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# Describing Function Approach with PID Controller to Reduce Nonlinear Action

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**ABSTRACT-** The nonlinear effect in the control system is so important and it may have a hard or soft effect on the electrical, mechanical, biological, and many other systems. This paper analyzes the describing function (DF) which is the transfer function of the nonlinear (NL) control systems of many NL elements found such as saturation, and backlash. The effect of the NL on the third-order delayed system is considered. The PID controller is considered the heart of the control system and continuously finds the error between input and output, and formulates the desired signal for the actuator to control the plant. Experimental tanning of PID controller with the saturation NL as a case study with buffer Operation Amplifier (Op-Amp) to maintain the gain and phase shift. In addition, a low pass filter (LPF) is used in the feedback to minimize and attenuate the effect of the NL in the closed-loop control system. The Fourier series is used to analyze the DF. The results show the effectiveness of the proposed algorithm.

**General Terms:** Ziegler Nichols, Approximation, Steady-state, Step response, Harmonics.

**Keywords:** Nonlinear, PID controller, Describing function, Fourier transform, Third-order delayed system.

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## 1. INTRODUCTION

The nonlinear (NL) phenomena, functioning is considered the main element in most control systems as they are inherently nonlinear. It could be either continuous or discrete signals. Normally, the systems behave as linear elements due to the approximation or the linearization in the specific range of operation or at low speed. Many nonlinear models represent real-world processes such as gravitational, robotics, coulomb friction, and electrostatic attraction, biomedical engineering, aircraft and voltage-current relationship in electrical systems [1]. The Phase plane method, describing function method, the small-gain theorem and Lyapunov method are the main techniques used in nonlinear systems [2].

For the prediction or measurement of the output signal response, it is useful to consider that the nonlinearity is linear in a small region of operation with small forcing inputs and NL is less cost

and better in behavior due to its small size compared to the linear systems [3].

Experimental analysis of the feedback control system in verification and testing of nonlinear models due to a variety of nonlinear sources used in the industry [4]. The Describing function (DF) is a well-known approach in the validation and analysis as an approximation method of the frequency response of nonlinear systems. The DF method is defined as the ratio of the fundamental output response to the amplitude of the sinusoidal input. The Fourier series first harmonic is considered to reduce the effects of harmonics of the higher frequencies [5]. The DF is one of the frequency response approximate methods used to predict jump phenomenon, oscillations characteristics, the limit cycles and, sub-harmonic when the NL system is subjected to sinusoidal, Gaussian noise and level inputs. Stability of a linear system and a static nonlinear using a Nyquist plot developed in [6] through a DF approach. Limit cycle and stability using Fuzzy logic in [7]. Stability analysis of NL systems with neural networks [8].

The main contribution of this paper is to design a PID controller to reduce the effects of relatively weak nonlinear with third-order delayed system.

PID controller is used to controlling more than 90% of control processes in the industry due to its functional simplicity, the ease and the performance of each part of the control i.e. the present, future and past control of the error [9]-[10].

The PID controller has based on either series (PI+PD) or parallel structure. The parallel structure is well-known in the literature of closed loop control systems.

Ziegler and Nichols method are used to tune the PID controller to minimize the quality criteria IAE, ITAE and ISE for the time-delayed control systems [11].

Different nonlinear elements are tested via a control process simulator with and without a PID controller to check the effectiveness of the proposed algorithm. Drift error and the compensation of the angle reduced satisfactory through the buffer unit and low pass filter (LPF) in the feedback. LPF will be developed to control the slope of gain around unity and phase between [0-1800]. This contribution of this paper utilized the benefits of a combined PID controller, nonlinear element and buffer operational amplifier to control the gain and the phase and LPF as well due to step and sinusoidal inputs.

The approach of selecting the PID controller parameters is to reduce as much as possible the effect of the nonlinear behavior of the introduced NL elements. Ziegler and Nichols tuning method PID controller is based on experimental step responses [12]. The future scope in the nonlinear reduction utilized is the fault detection and tolerance using neural network, machine learning [13]. Fuzzy logic genetic algorithm used for the noise reduction of nonlinear behavior in the system and considered as a recent approach in this field. An Estimator based inverse dynamics controller (EBIDC) with Artificial Neural Network (ANN) based state estimation scheme for nonlinear with disturbances and measurement noises that are stochastic in nature and dynamic window function having multi spectral characteristics are used as well [14]-[15].

The rest of the paper is organized as follows: *Section 2* introduces the principles of DF. *Section 3* will introduce the nonlinear elements. *Section 4* will introduce the PID controller. Conclusions will be introduced in *Section 6*.

## 2. DF THEORY

The DF is established in 1930 Nikolay M Kreyol and Nikolay Googlebot. The frequency response can be defined as a steady state response with sinusoidal input. Let

$$x(t) = X \sin(\omega t)$$

Fourier series is used to analyze the behavior of the DF and can be expressed as in the following equations. The output response  $y(t)$  is:

$$y = Y_0 + A_1 \cos \omega t + B_1 \sin \omega t + A_2 \cos 2\omega + B_2 \sin 2\omega \tag{1}$$

For a symmetrical NL waveform around the origin,  $Y_0$  is the DC level or the average is zero,

$$y = A_1 \cos \omega t + B_1 \sin \omega t + A_2 \cos 2\omega t + B_2 \sin 2\omega t + \dots \tag{2}$$

As mentioned earlier only fundamental harmonics are needed in the DF approach. Hence,  $y_1$  need only be considered [16].

$$y_1 = A_1 \cos \omega t + B_1 \sin \omega t \tag{3}$$

‘Or’  $y_1(t) = Y_1 \sin(\omega t + \phi_1)$

In phasor form

$$Y_1 \angle \phi_1 = B_1 + jA_1 = \sqrt{B_1^2 + A_1^2} \angle \tan^{-1} \left( \frac{A_1}{B_1} \right) \tag{4}$$

Fourier series coefficients (A1 and B1) are given by:

$$A_1 = \frac{1}{\pi} \int_0^{2\pi} y \cos \omega t d(\omega t) \tag{5}$$

$$B_1 = \frac{1}{\pi} \int_0^{2\pi} y \sin \omega t d(\omega t) \tag{6}$$

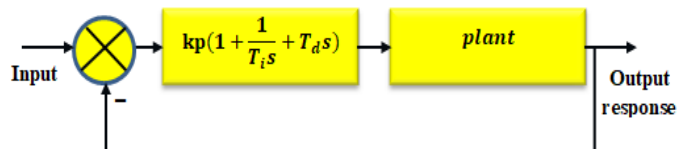
The DF (N) was invented as the complex ratio of fundamental harmonics with its angle to the amplitude of the input.

$$N = \frac{Y_1}{X} \angle \phi_1 = \frac{\sqrt{A_1^2 + B_1^2}}{X} \angle \tan^{-1} \left( \frac{A_1}{B_1} \right) \tag{7}$$

## 3. DF FOR SOME NONLINEARITIES

In the analysis of the NL, there are many assumptions to simplify the analysis DF such as, the NL elements are time-invariant like the linear systems and if there are more than one NL element it should be combined as one NL odd function.

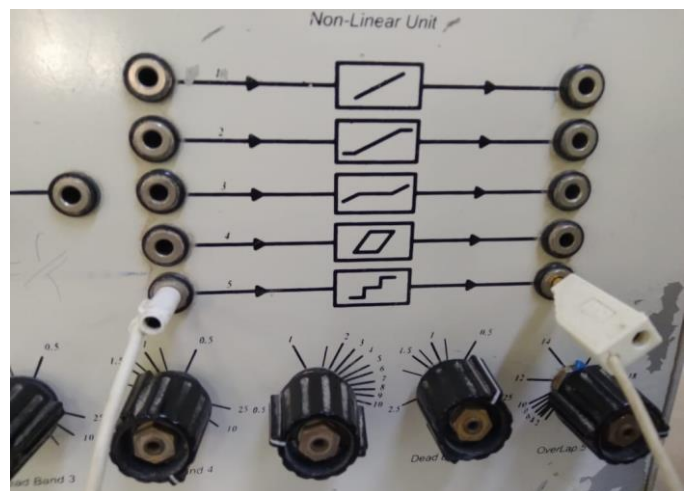
The classical structure of the closed-loop control system in the general form is shown in *figure 1*.



**Figure 1.** Closed-loop system

To verify these assumptions, the next linear element has an LPF performance specification.

The possibility of the NL elements can be found in the front panel of the control system [*figure 2*].



**Figure2.** Nonlinear elements in the real world

The input / output response of the first element (top) in *figure 2* is shown in *figure 3*.

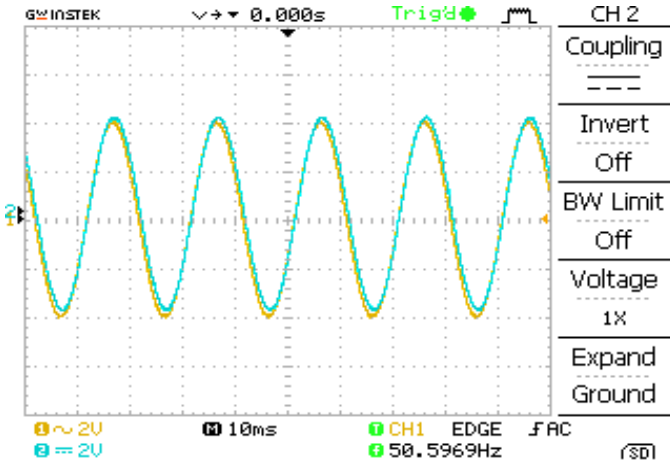


Figure3. Input/ output response of the first element

### 3.1 Nonlinear Unit with Saturation

In the DC motors, the magnetizing curve is considered as a good example of Saturation NL. The characteristic curve for saturation is shown in figure 4.

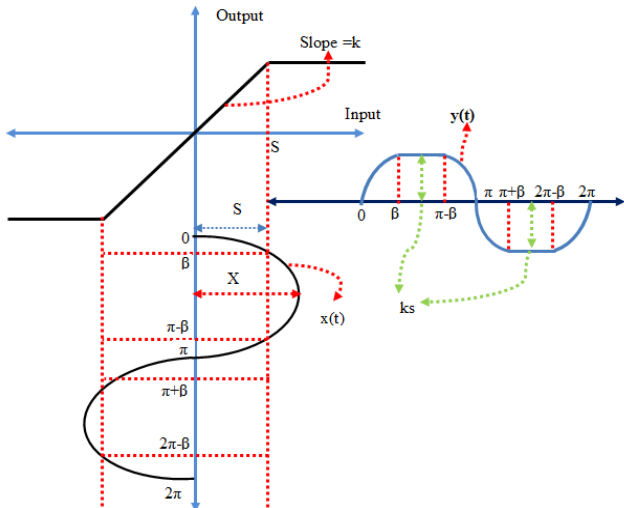


Figure4. Characteristics of the saturation nonlinear [17]

The DF of the saturation NL which is shown in the second unit of the NL elements (figure2) can be derived as follows [18].

$$N = \frac{B_1}{X} \angle 0 \quad (8)$$

$$N = \frac{2k}{\pi} \left[ \sin^{-1} \left( \frac{s}{X} \right) + \frac{s}{X} \sqrt{1 - \left( \frac{s}{X} \right)^2} \right] \angle 0 \quad (9)$$

$$B_1 = \frac{2kX}{\pi} \left[ \sin^{-1} \left( \frac{s}{X} \right) + 2 \frac{s}{X} \operatorname{cossin}^{-1} \left( \frac{s}{X} \right) - \operatorname{sinsin}^{-1} \left( \frac{s}{X} \right) \operatorname{cossin}^{-1} \left( \frac{s}{X} \right) \right] \quad (10)$$

$$N = \frac{2kX}{\pi} \left[ \sin^{-1} \left( \frac{s}{X} \right) + \frac{s}{X} \sqrt{1 - \left( \frac{s}{X} \right)^2} \right] \quad (11)$$

Where  $k$ ,  $s$ ,  $X$  is the slop, the level of the saturation nonlinear and amplitude of the sinusoidal input as in figure 4. Applying sinusoidal input from the function generator on the nonlinear saturation individually results in the following figure and its same procedure for the rest at the beginning of the analysis.

The actual input /output response of this type of NL is shown in figure 5.

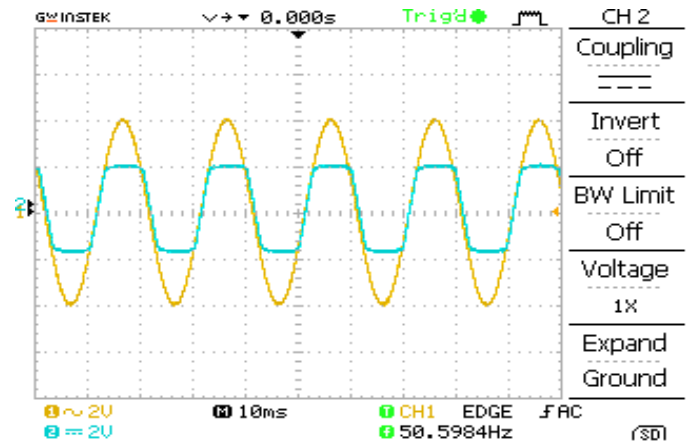


Figure5. Input /output of the saturation NL

### 3.2 Backlash Nonlinear

The characteristics curve of the backlash NL is shown in figure 6.

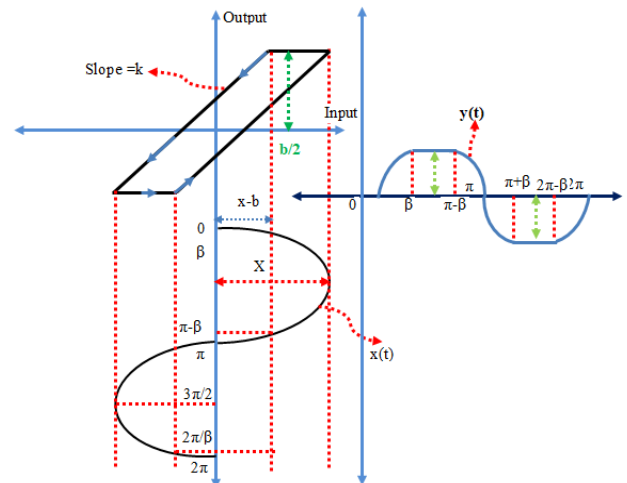


Figure6. Characteristics of the backlash NL [18]

The actual input/output figure of the backlash NL element is shown in figure7.

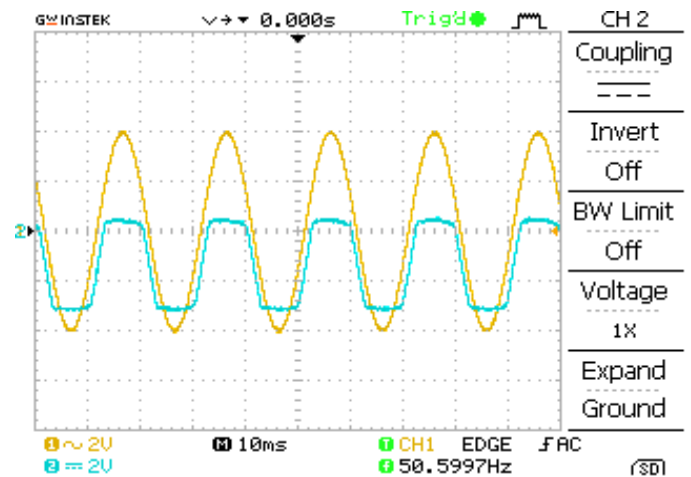


Figure7. Input/output relationships of the backlash unit

The derivation of the DF of the backlash is performed as follows [30]:

$$A_1 = \frac{2}{\pi} \int_0^\pi y(t) \cos(\omega t) d\omega t \quad (12)$$

$$A_1 = \frac{2}{\pi} \int_0^{\pi/2} (X \sin \omega t - \frac{b}{2}) y(t) \cos(\omega t) d\omega t + \frac{2}{\pi} \int_{\pi/2}^{\pi-\beta} (x - \frac{b}{2}) y(t) \cos(\omega t) d\omega t + \frac{2}{\pi} \int_{\pi-\beta}^\pi (X \sin \omega t + \frac{b}{2}) y(t) \cos(\omega t) d\omega t = \frac{x}{\pi} \cos^2 \beta \quad (13)$$

$$B_1 = \frac{2}{\pi} \int_0^\pi y(t) \sin(\omega t) d\omega t \quad (14)$$

The DF is

$$N = \begin{cases} 0, & x < b/2 \\ \sqrt{\left(\frac{A_1}{x}\right)^2 + \left(\frac{B_1}{x}\right)^2} \tan^{-1} \frac{B_1}{A_1} & x \geq b/2 \end{cases} \quad (15)$$

### 4. PID CONTROLLER

In the real world, most the systems are modeled as a second-order or simplified second-order system. The plant is a physical object to be controlled. The parameter variations of the actual system need to be settled via a PID controller to improve the system response [19]-[20]. A parallel PID compensator adjusted by Ziegler–Nichols is designed but doesn't apply directly to all system structures of PID controller either for linear or NL systems [18].

The design of the PID controller parameters (kp, ki, kd) is to be able for disturbance rejection [21]-[22].

Besides the simple structure, the advantages of the PID controller are:

1. Reduce the rise time via proportion action
2. Eliminate the steady state error with integral action
3. Improve the accuracy with the derivative action [10]-[24].

Normally, DF analysis is utilized for indicating compensation approaches and the possibility of elimination of signal oscillations. The PID controller used in this work is shown in figure8.

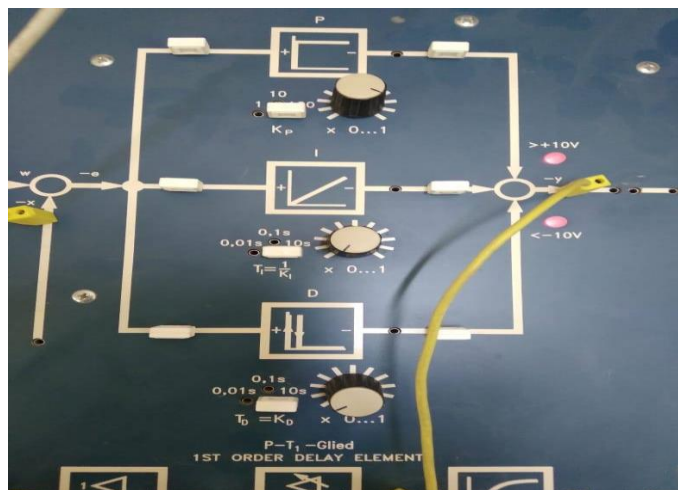


Figure8. PID controller

The relationship between the DF of the dead zone and the saturation nonlinearities at the beginning of the real operation is [18]:

$$N_{dead\ zone} = k - N_{saturation} \quad (16)$$

### 5. PROPOSED ALGORITHM

The general structure of the closed-loop system with non-linear element containing NL element as an actuator. This can be illustrated in figure 9.

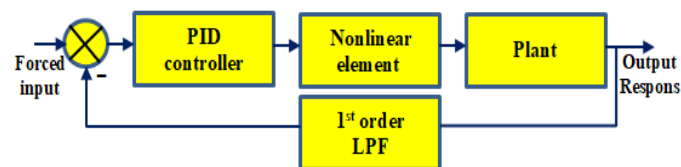


Figure9. Block diagram of the closed-loop system

In this work, a system of 3rd order with delay is considered. The simplest type of delay to visualize is the pipeline delay. The third-order exponential delay equation is called delay [25]. Temperature control is another type of delayed systems. The simulated electrical 3rd order is obtained as in figure10.

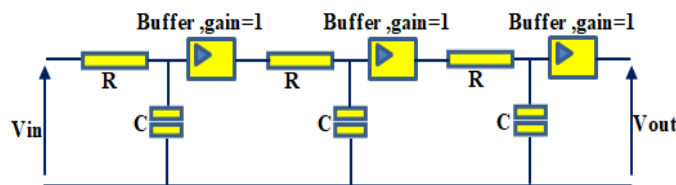


Figure10. Simulation of the 3rd order delayed system

For square waveform as a generated step input, 3<sup>rd</sup> order input /output waveform is shown in the figure11.

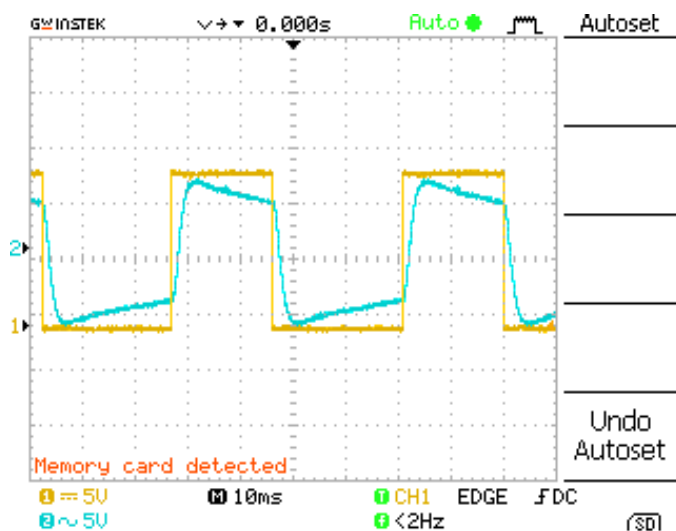
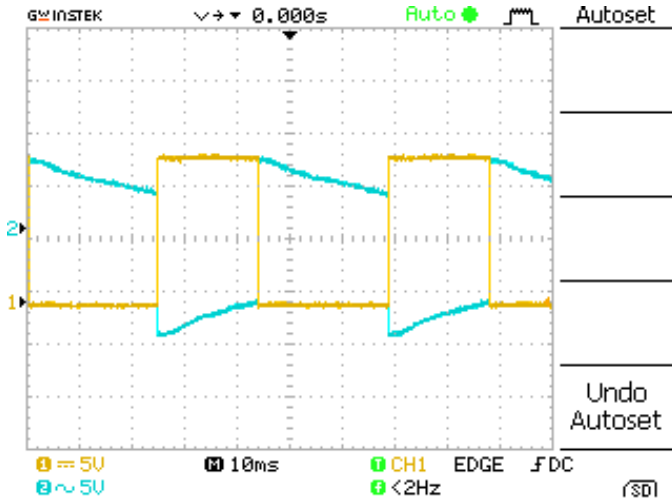


Figure11. Input /output relationship of the 3rd order system

To get sufficient operation between the delayed periods a buffer of unity gain and phase shift of 180 is used [23]. The output response of this buffer with its input is shown in figure 12.



**Figure12.** Amplitude and phase shift of the buffer unit

There are two common formulas used to describe the PID control algorithm. The first formula as in [26]:

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d(\tau) + k_d \frac{de(t)}{dt} \quad (17)$$

Where:

$u(t)$  is the output of the controller, fed into the process.

$e(t)$  is the error between input and output.

$k_p, k_i, k_d$  are the respective P, I, and D constants

A second formula of the PID equation is:

$$u(t) = k_p(e(t) + \frac{1}{T_i} \int_0^t e(\tau) d(\tau) + T_d \frac{de(t)}{dt}) \quad (18)$$

Where

$T_i, T_d$  is the integral time and derivative time respectively.

$$k_i = k_p * \frac{1}{T_i} \quad (19)$$

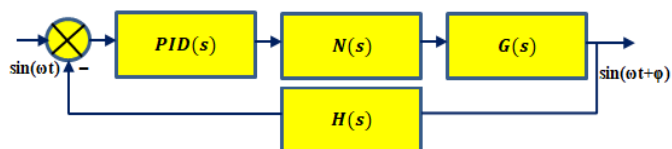
$$k_d = k_p * T_d \quad (20)$$

The reasons for using equation (18) are:

Using the time constants  $T_i$  and  $T_d$  in real-time applications [27].

The proportional gain ( $k_p$ ) is considered as the PID controller overall gain [28]. It's easy to use Ziegler-Nichols tuning tables

The closed-loop Transfer function of the system is shown in figure13.



**Figure13.** Closed loop structure of nonlinear with sinusoidal input

For a negative feedback system, the general form of the closed loop transfer function is

$$T(s) = \frac{G_{pid}(s)NG(s)}{1+G_{pid}(s)NG(s)H(s)} \quad (21)$$

The transfer function of the 3rd order delayed system as in eq. (22).

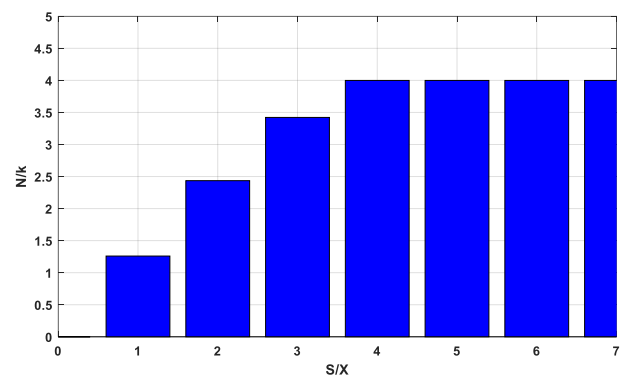
$$G(s) = \frac{1}{1x10^{-9}s^3 + 3x10^{-6}s^2 + 3x10^{-3}s + 1} e^{-0.4s} \quad (22)$$

This is for  $R=1k\Omega$  and  $C=1\mu F$  and a delay time of 0.4 sec.

$$N = 2.546k [\sin^{-1}(\frac{S}{4}) + \frac{S}{4} * \sqrt{1 - (\frac{S}{4})^2}] \quad (23)$$

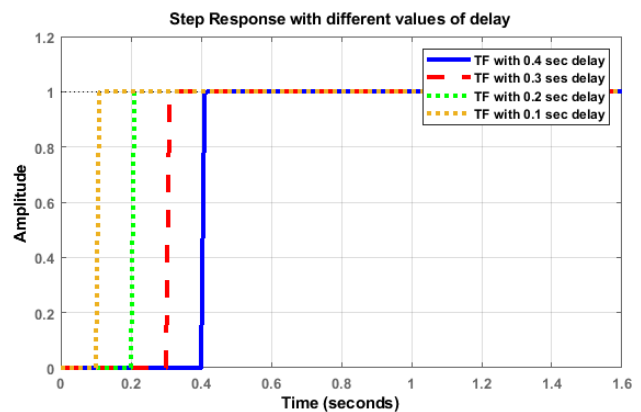
$$N/k = 2.546 [\sin^{-1}(\frac{S}{4}) + \frac{S}{4} * \sqrt{1 - (\frac{S}{4})^2}] \quad (24)$$

The plot of  $N/k$  against  $S/X$  [ $X$  is the amplitude of the sinusoidal input = 4V] as in figure 14.



**Figure14.** N/k versus S/X characteristics

The step response of the complete system with different values of delay time is performed as in figure 15.



**Figure15.** Step response of the system

From the figure 14 and figure 15, it is possible to see the three properties of this DF [29]-[30]:

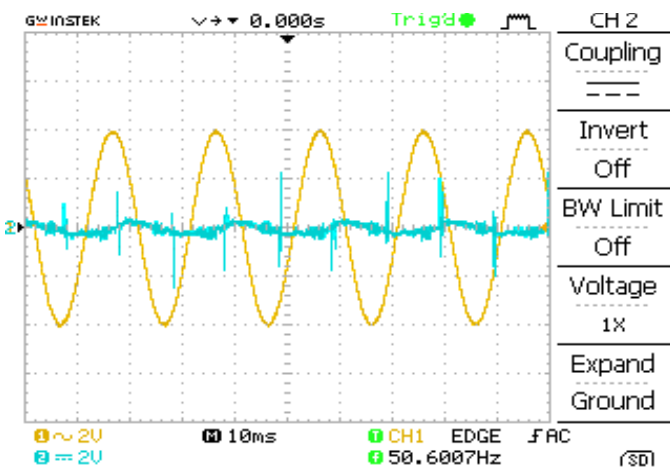
1.  $N= k$  in the linear region and the saturation will be a very small effect or even not occur.
2. Inversely proportional between input amplitude and DF, this will reduce the gap between the input and the output.
3. Steady state error is zero directly after the delay time.

The robustness parameters test is performed to validate the design using MATLAB/Simulink. The performance specifications of this test are listed as in table 1.

**Table 1.** Performance and robustness parameters

Performance and Robustness	
	Tuned
Rise time	5.42e-20 seconds
Settling time	4.91 seconds
Overshoot	2.22e-14 %
Peak	1
Gain margin	Inf dB @ NaN rad/s
Phase margin	-90 deg @ 7.97e+07 rad/s
Closed-loop stability	Stable

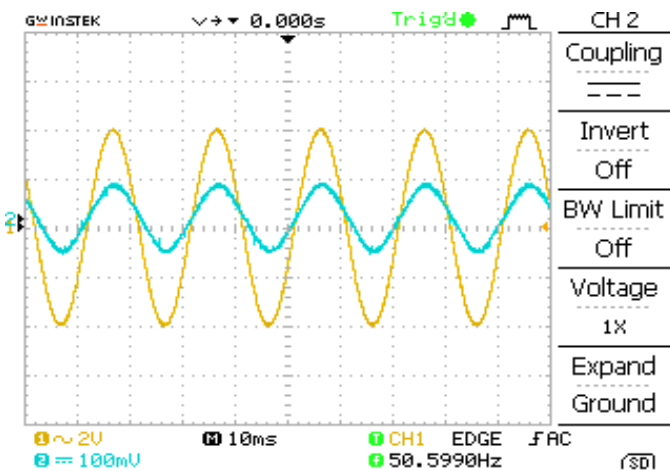
The closed loop with saturation NL system as a case is checked first without any controller. The input/output relationship is shown in figure 16.



**Figure16.** Relationship between input/output of the system containing saturation

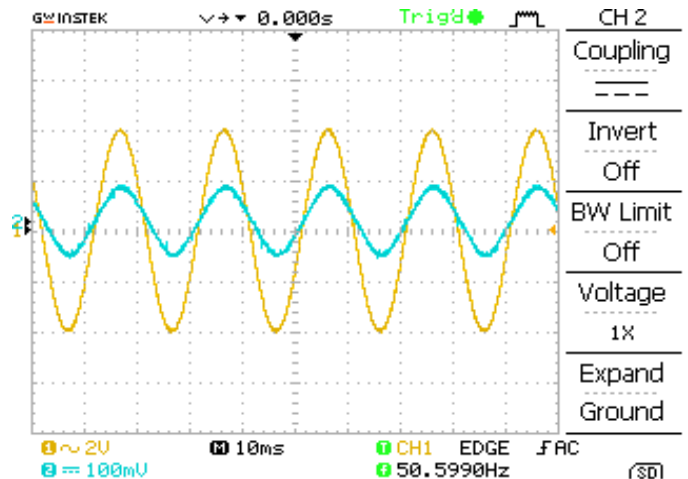
Then, waveform output with PI controller also couldn't control the output to be as a sinusoidal input.

The input/output of the system containing saturation nonlinear with PI controller is shown in figure 17.



**Figure17.** Closed loop response with PI controller

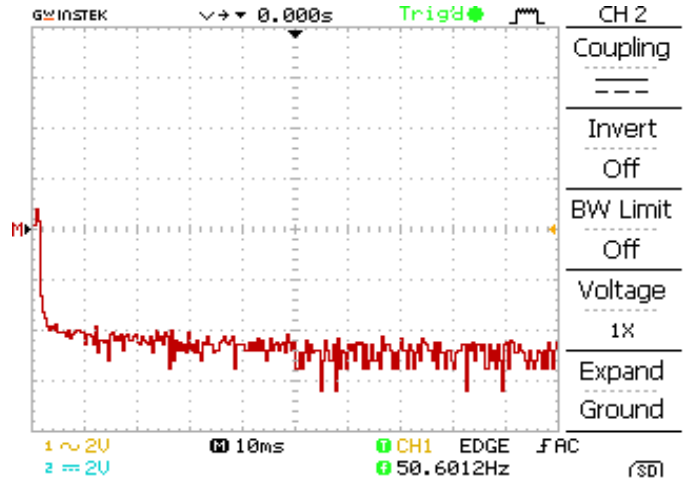
Derivative action is added to the PI controller to complete the PID controller. In the beginning, the response is in figure 18.



**Figure18.** Saturation NL without fine-tuning of PID

PID controller can be utilized due to the controller's effectiveness in a wide range of operation conditions [31]-[32].

The Fourier transform of this case is shown in figure 19.



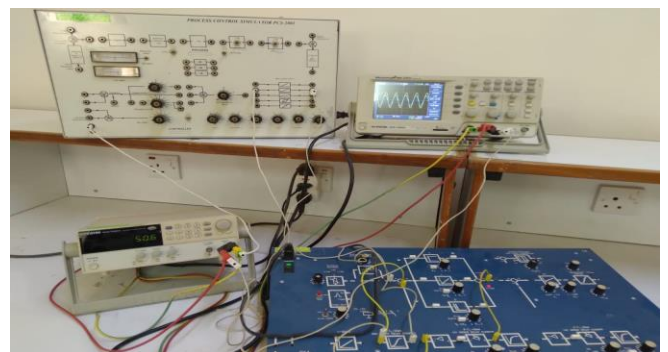
**Figure 19.** Fourier transform in the first case

The output response after fine tuning of the  $k_p, k_i, k_d$  depends on  $k_p=10, k_i=0.2, k_d=4$ .

The PID controller transfer function is developed as in eq. (25)

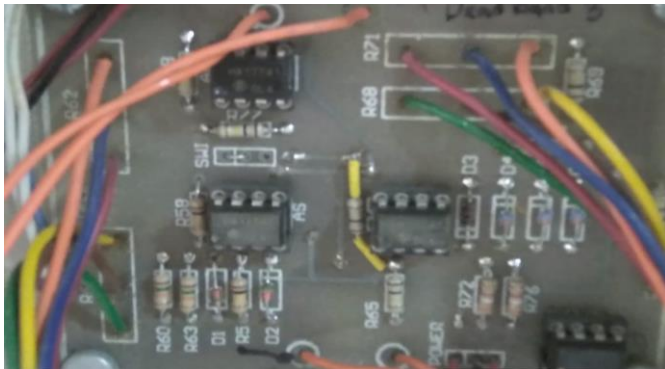
$$G_{pid}(s) = \frac{0.8s^2 + 2s + 1}{0.2s} \quad (25)$$

The complete setup is shown in figure20.



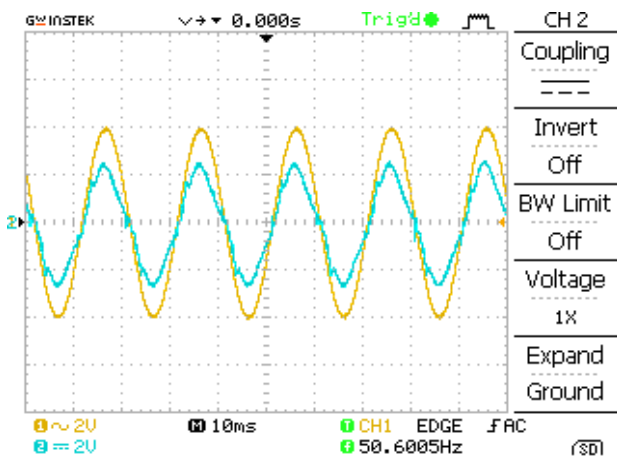
**Figure20.** Complete setup of the closed-loop system

The HA17741 is used as the main part of the NL development as internal phase compensation with high-performance op-amp [28]. The hardware implementation of this circuit is shown in *figure 21*.



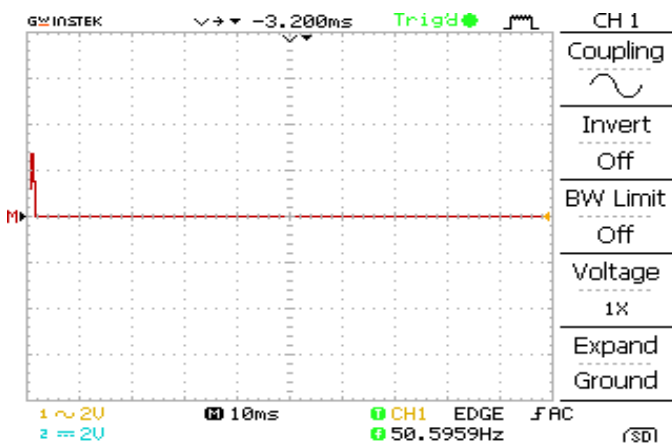
**Figure21.** Hardware nonlinear circuit

The final response of the closed-loop system containing saturation NL with fine tuning of PID controller is shown in *figure22* and the best response recorded as in *figure19* but unfortunately, the figure was deleted from the memory due to inconvenient action.



**Figure22.** The final output response with the PID controller

The Fourier transform of this system is shown in *Figure23*. This ensures that the system has no phase shift and the system is stable.



**Figure23.** Fourier transforms of the system

## 6. CONCLUSIONS

This paper performed a DF for a NL system containing one NL element such as saturation, backlash, dead zone, etc. Third order delay system is described as a plant with saturation NL element as a soft actuator. Frequency response is obtained with sinusoidal and step input generated experimentally. The Fourier series is a powerful tool to analyze the DF. PID controller as a linear part of the closed-loop control system was able to reduce the effect of the NL precisely. The NL systems are considered as linear when the DF is equal to the variation of the NL characteristic in the specified range of operation. Assumptions in the evaluation of DF are so important such as one NL element or sinusoidal input to complete the algorithm sufficiently. The intelligent Fuzzy PID controller, neuro-fuzzy controller and adaptive control can be used as a future scope.

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